Adaptive Power Allocation for Noncooperative OFDM Systems in UWA Interference Channels

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Abstract—Noncooperative underwater acoustic (UWA) communication systems are prone to interfere with each other since the limited resource offered by the channel is not regulated by any standard. Mutual interferences can significantly degrade the performances of such systems and it is necessary to find policies allowing UWA devices to access the same physical resource. In this paper, we consider noncooperative UWA OFDM systems competing to access the same portion of the spectrum. We show that an efficient decentralized power allocation strategy can be achieved when all the communication links apply a waterfilling policy only based on little knowledge about their environment. Numerical simulations performed with real UWA channels sounded at-sea demonstrate the relevance of our approach.

Index Terms—Underwater acoustic communications, power allocation game, interference channel, OFDM.

I. INTRODUCTION

In the absence of spectrum regulation, underwater acoustic (UWA) communication systems may suffer from unintentional jamming due to interference from other transmitters [1], [2]. Mutual interferences between two or more sound sources appear when they operate in nearby geographical areas, are active simultaneously and transmit over the same frequency band. Dealing with interferences is not an easy task for UWA systems because they cannot precisely know the characteristics of the interfering signals. Due to the very limited available bandwidth, a fixed spectrum assignment for specific applications is not conceivable. A common approach to make UWA communications more robust is to mitigate external interferences at reception [1], [2]. An alternative or complementary approach is to design smarter transmitters with the capability of adapting their transmission parameters to the soundscape in which they operate. Such an optimization depends on some performance metric that has to be determined by the system designer, and is made possible only if the transmitter can retrieve some information on the link quality. This is usually made via feedback from its intended receiver [3].

In a recent work [4], we studied the scenario where several noncooperative UWA transmitter-receiver pairs compete to get access to the same physical channel. Each acoustic transmission is then seen by the other communication links as external interference. To limit mutual interferences, we proposed a decentralized power allocation strategy based on game theory and on an OFDM modulation scheme. Without cooperation and centralized control, we showed that efficient spectral power allocations can be achieved when each transmitter selfishly seeks to maximize a performance metric related to its information rate. The optimal allocation is shown to be a waterfilling solution only based on the knowledge of the channel statistics as well as the noise plus interference average power.

The purpose of this paper is to experimentally validate the analysis provided in [4] in a new context thanks to real channels recorded in the Mediterranean Sea. The impact of estimation errors on the channels statistics required by the proposed power allocation strategy is also studied.

The paper is organized as follows. The main theoretical results of [4] are summarized in Section II. Application of these results to real data are presented in Section III, followed by conclusions in Section IV.

Notations: Uppercase and lowercase boldface letters, e.g. \( \mathbf{A}, \mathbf{x} \), denote matrices and vectors, respectively. The superscript \(^T\) denotes transposition. \( [x]^+ \) is equivalent to \( \max(0, x) \). Finally, \( \mathbb{E}\{\cdot\} \) denotes expectation.

II. POWER ALLOCATION STRATEGY

A. System model

Following [4], we consider a finite set \( \mathcal{M} = \{1, \cdots, M\} \) of OFDM links (the players of the game) sharing the same physical resource. Each transmitter uses \( N \) subcarriers whose allocated power can be adjusted. The total bandwidth is \( B = N\Delta f \) with \( \Delta f \) the subcarriers spacing, and the OFDM symbol duration is \( T = T_s + T_g \) with \( T_s = 1/\Delta f \) and \( T_g \) the cyclic prefix duration chosen greater than the channel delay spread. The strategy space of transmitter \( i \in \mathcal{M} \) is defined as its possible power allocation set, i.e.

\[
P_i \triangleq \left\{ \mathbf{p}_i \in \mathbb{R}_+^N : \sum_{n=1}^N p_i(n) \leq P_i^{\max} \right\},
\]

(1)

where \( \mathbf{p}_i = [p_i(1), \cdots, p_i(N)]^T \) and \( p_i(n) \) is the power allocated by transmitter \( i \) on the \( n^{th} \) subcarrier. Each player competes rationally and seeks to maximize a metric called utility function that describes its information rate. The low speed of sound underwater combined with the rapidly time-varying nature of the medium prevent short period feedback policies from being implemented, since it would result in outdated channel estimates at the transmitter side. Therefore, the performance metric can only be related to some “average” information rate. In addition, each transmitter can only update...
its power spectral density (PSD) according to long term statistics on their direct channel and on the overall noise plus interference computed by their respective receivers. Thus, based on [4, Sec. II-B], the utility function that each player \( i \) wants to maximize is here defined as

\[
    u_i(p) = \frac{1}{NT} \sum_{n=1}^{N} \log \left( 1 + \gamma_i(n)p_i(n) \right),
\]

where \( p \in [p_1, \ldots, p_M] \) is the strategy profile aggregating the power allocations of all the links and

\[
    \gamma_i(n) = \frac{g_i(n)}{\sigma_{w_i}^2(n) + \sum_{j \neq i} \|h_{ji}(n)\|^2} p_j(n),
\]

with \( h_{ii}(n) \sim CN(\mu_{ii}(n), \sigma_{ii}^2(n)) \) the direct subchannel \( n \), \( h_{ji}(n) \) the interference subchannel \( n \) between transmitter \( j \) and receiver \( i \) and \( \sigma_{w_i}^2(n) \) the noise variance at the subcarrier \( n \). The term \( \gamma_i(n)p_i(n) \) in (2) can thus be understood as an averaged signal to noise plus interference ratio (SINR), weighted by the function

\[
    g_i(n) = \frac{K_i(n)}{K_i(n) + 1} e^{-\text{Ei}(-K_i(n))},
\]

where \( K_i(n) = |\mu_{ii}(n)|^2/\sigma_{ii}^2(n) \) is the Rice factor of subchannel \( h_{ii}(n) \), with \( \mu_{ii}(n) \) and \( \sigma_{ii}(n) \) its mean and standard deviation, and \( \text{Ei}(x) \) denotes the exponential integral function, for \( x > 0 \), as

\[
    \text{Ei}(x) = -\int_{x}^{\infty} \frac{e^{-t}}{t} \, dt.
\]

Depending only on channel and noise plus interference statistics, \( u_i(p) \) is in agreement with the previous system constraints. The game that models the competitive access to the UWA channel is then defined as the triplet:

\[
    G = \left\{ M, \{p_i\}_{i=1}^{M}, \{u_i\}_{i=1}^{M} \right\}.
\]

### B. Iterative waterfilling

Based on the game \( G \), the power allocation strategy is obtained by finding for each player \( i \in M \) the optimal vector \( p_i^* \in P_i \) that maximizes its utility function \( u_i(p_i(p), p)^{-1} \), given that other players are also playing their optimal strategies denoted by \( p_{-i}^* = [p_1^*, \ldots, p_{i-1}^*, p_{i+1}^*, \ldots, p_M^*] \). These optimal power allocations are reached non-cooperatively, each link treating the interferences caused by others as noise. Such a strategy profile \( p^* = [p_1^*, \ldots, p_M^*] \), where no player has an interest to deviate from, is called a Nash Equilibrium (NE) of the game [7].

The existence of Nash equilibria for game \( G \) is discussed in [4]. It is also shown that, at a NE, each transmitter allocates its power by waterfilling on every subcarrier according to the direct channel and interference plus noise statistics that its corresponding receiver has fed back, i.e.,

\[
    p_i^*(n) = \left[ \frac{1}{\lambda_i} - \frac{1}{\gamma_i(n)} \right]_{+},
\]

where \( \lambda_i \) is chosen to satisfy the power constraint (1) with equality. In practice (and under specific conditions), a NE of game \( G \) is reached using iterative algorithms [8].

In (7), it is implicitly assumed that each receiver is able to perfectly estimate the average power of noise plus interference as well as the statistics of its direct channel. This assumption will be relaxed in Section III-C.

### III. EXPERIMENTAL RESULTS

#### A. Experiment setup

The proposed decentralized power allocation strategy is here illustrated with real channel measurements recorded off the coast of Toulon, France, in July 2015. Three transmitter-receiver pairs were deployed according to the configuration depicted in Figure 1. Three hydrophones RX1, RX2 and RX3 were immersed at depths of 4 m, 12 m and 16 m respectively, at the same fixed location. The channels from one transmitter to the receivers were sounded at three different locations, corresponding to TX1, TX2 and TX3. The signal to noise ratio (SNR) for each link were approximately 15 dB, 12 dB and 9 dB from the nearest to the farthest transmitter.

Measurements of the channel impulse responses were obtained by successive matched filtering to a known probe signal transmitted repeatedly. The probe signal used during the experiments was a m-sequence of 511 BPSK chips transmitted at a symbol rate of 8.7 kbd. Such a sequence can capture arrivals delayed up to 58 ms and channel estimates can be updated up to 17 times per second. The channel sounding duration was \( T_{s} = 25 \) seconds. Measurements were made at a carrier frequency of 10.5 kHz and time-varying Doppler shifts were mitigated by the iterative resampling procedure presented in [9], [10]. The processing gain offered by the m-sequence is 27 dB. The measured channels are thus considered as the ground-truth from the point of view of the multiuser system simulated next. Consequently, their Rice factors and averaged frequency responses are said to be the perfectly estimated statistics in the following. The channels frequency responses in a 6 kHz bandwidth, averaged over the sounding duration, are shown in Figure 2. The Rice factors \( K_i(n) \) for the direct frequency subchannels are between 1.4 dB and 6 dB in average, depending on the link.

Based on these channel statistics, the behavior of three OFDM UWA links with \( N = 256 \) subcarriers is simulated. All the links use the same bandwidth \( B = 6 \) kHz centered around \( f_c = 10.5 \) kHz with the power constraint \( P_i^{\text{max}} = N \).
Figure 2. Averaged frequency responses over time of the channels sounded at sea, $T_{obs} = 25s$. 

The symbol time is $T = 57.7$ ms with a cyclic prefix time $T_g = 15$ ms. The game is run during 30 iterations. At time $t = 0$, we assume that all the links play the uniform power allocation strategy. As the game progresses, the players update their strategy one after the other according to the waterfilling policy in (7). The long term direct channel statistics required by the transmitter are supposed to be estimated and fed back by its corresponding receiver. The duration over which these statistics are computed is chosen equal to the channels sounding duration $T_{obs}$. It also corresponds to the period at which each player updates its strategy. The next subsection presents the results for the case of perfectly estimated channel statistics, while Section III-C analyses the impact of estimation errors on the power allocations.

B. Perfectly estimated channel statistics

Figure 3 shows the last power allocation of the game as a bar graph where the powers allocated by the three players are stacked on each subchannel. This result is in agreement with the direct channel average frequency responses shown on the diagonal of Figure 2. The third link, for instance, has more incentives to use the subchannels in the 7.5 to 9.5 kHz band, since it experiences its best channel gain (see $|h_{33}(f)|^2$ on Figure 2). Despite its channel is fairly good even up to 10.5 kHz, this player is not inclined in allocating its power around these frequencies, since on the one hand the interference channels gains $|h_{131}(f)|^2$ and $|h_{23}(f)|^2$ are high and on the other hand, the direct channels $|h_{11}(f)|^2$ and $|h_{22}(f)|^2$ are also good for player 1 and 2. Indeed, as they have more interest to use these subcarriers, it leads to high interference on player 3. Being the closest to the three receivers, transmitter 1 is inclined to selfishly use the whole 6kHz bandwidth to its own benefit, allocating more power on its best subchannels and letting the two other players share those where its has allocated less or no power.

The evolution of the utility functions $u_i(p)$ (2) over the game repetitions is depicted in full line in Figure 4. At the end of the game, all the players have multiplied their utility by at least two compared to the initial uniform PSD and are close to a strategy where none of them is inclined to deviate. For the links 1 and 2, more than 95% of the final utility is reached in less than 8 iterations. In [4], the same amount of the final utilities is attained in 5 iterations. The slower convergence toward a NE is explained by a higher number of players combined with the sequential nature of the update scheme [11].

C. Imperfect estimation of the channel statistics

In practice, the OFDM receivers often have to estimate the direct channels with pilot symbols. Any estimator produces errors and the channel statistics cannot be perfectly known. Thus, the actual waterfilling strategy can be rewritten as

$$\hat{p}_i(n) = \left[ \frac{1}{\lambda_i} - \frac{1}{\hat{\gamma}_i(n)} \right]^+$$

(8)
where \( \hat{\gamma}_i(n) \) is an estimator of \( \gamma_i(n) \) defined in (3). The same simulations are run considering some uncertainty on the channel statistics conveyed by the variance of the estimator. The nature of the errors produced by any estimator depends on its implementation. In order to avoid system-specific concerns, we assume an unbiased \( \hat{\gamma}_i(n) \) having its variance \( \sigma^2_{\hat{\gamma}_i}(n) \) lower bounded by the Cramér-Rao Bound, denoted by \( I^{-1}(\gamma_i(n)) \). The game is now simulated with the three UWA OFDM links playing successively their waterfilling strategy based on the estimator \( \hat{\gamma}_i(n) \) modeled as a gaussian random variable with mean \( \gamma_i(n) \) and variance \( \sigma^2_{\hat{\gamma}_i}(n) = R \times I^{-1}(\gamma_i(n)) \), for different values of \( R \).

Figure 4 shows the resulting utilities of this new game with \( R = 1 \) and 10 along with the case of perfect knowledge. As the intuition could suggest, the utility functions of the players decrease with the estimators efficiency. The impact of errors is lesser for player 2 and 3, their disadvantageous situation relatively to the transmission geometry being preponderant. If the estimator has minimal variance \( (R = 1) \) the errors do not significantly impact the utility functions. Most importantly, even if the estimator is inefficient \( (R = 10) \), the resulting utilities are still almost twice higher than the initial uniform PSD.

Figure 5 shows the last power allocation for the game with imperfect knowledge of \( \gamma_i(n) \) and \( R = 10 \). Compared to the perfectly known channel statistics, the three players are prone to simultaneously use the same subchannels. As the estimators variance \( \sigma^2_{\hat{\gamma}_i}(n) \) becomes higher, their strategies are less correlated with the overall channel state. As a result, the links are inclined to interfere with each other, decreasing their utilities.

IV. CONCLUSION

A decentralized power allocation policy for UWA OFDM systems has been proposed. Numerical results using UWA channels sounded at-sea have shown that our approach can allow noncooperative UWA communication systems to share the available spectrum more efficiently, in an adaptive manner and only with little and possibly erroneous knowledge about their acoustic environment. Based on long term channel statistics, the proposed optimization metric copes with the low speed of sound and the rapidly time-varying nature of the medium and allows the implementation of slow feedback. In the light of these results, we believe that game theoretic tools can provide a novel way to consider multiuser UWA communications and networks, as it is already the case for terrestrial communications.

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